Mathematics

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DEVELOPMENT AND OPTIMIZATION OF A MATHEMATICAL INVENTORY MANAGEMENT MODEL UNDER DEMAND UNCERTAINTY

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ABSTRACT

Inventory management under uncertain demand requires decisions that balance ordering and holding costs against shortage risk and customer-service requirements. This article develops a mathematical framework for selecting an order quantity and reorder point when demand varies over time and replenishment lead time may expose the system to stockouts. The proposed framework combines a baseline economic order quantity relationship with a reorder-point policy, safety-stock logic, expected shortage or service-level measures, and scenario-based sensitivity analysis. The model is designed to be adaptable to empirical demand records, while clearly distinguishing analytical results from illustrative calculations. A reproducible workflow is provided for parameter estimation, optimization, and validation. Since no organization-specific demand, cost, or lead-time dataset was supplied, the numerical example and figure are explicitly illustrative and are not presented as empirical findings. The contribution is a transparent modelling structure that can be calibrated and tested for a specific inventory system before operational adoption.

KEYWORDS: Inventory Management, Demand Uncertainty, Mathematical Modelling, Economic Order Quantity, Reorder Point, Safety Stock, Optimization, Service Level

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INTRODUCTION

Inventory systems connect procurement, storage, production, and customer fulfilment. Excess inventory ties up working capital and increases holding, insurance, handling, and obsolescence costs, whereas insufficient inventory can cause lost sales, delayed production, emergency purchasing, and reduced service performance. The central decision problem is therefore not simply to minimize stock, but to choose replenishment decisions that achieve an appropriate balance between cost, availability, and operational risk.

The classical economic order quantity (EOQ) model provides a useful baseline by balancing ordering cost against cycle-stock holding cost under simplifying assumptions. In many practical settings, however, demand is variable, lead time is non-zero, and a replenishment decision must account for uncertainty during the period between placing and receiving an order. A reorder point and safety stock can extend the baseline model, while stochastic or scenario-based methods can represent uncertainty more explicitly. Standard inventory texts describe these relationships and their assumptions [1,2].

This article proposes a coherent mathematical framework for inventory systems in which demand uncertainty is represented through estimated demand distributions or scenarios. The framework is intended as a model-development contribution: the equations define the decision structure, and the validation plan specifies how the model should be calibrated against actual records. It does not claim empirical superiority until a relevant dataset and benchmark comparison have been completed.

2. AIM AND OBJECTIVES

The primary aim is to develop a mathematical inventory management model that supports order-quantity and reorder-point decisions under demand uncertainty while considering total relevant cost and service performance.

- To define the decision variables, parameters, assumptions, and operating constraints of the inventory system.
- To formulate an objective function incorporating ordering, holding, and shortage-related costs.
- To incorporate demand variability during replenishment lead time through safety stock or scenario-based risk measures.
- To identify a solution procedure for selecting order quantity and reorder point.
- To establish a sensitivity and validation framework for comparing the proposed model with a baseline inventory policy.

3. LITERATURE REVIEW AND CONCEPTUAL BACKGROUND

EOQ remains a foundational deterministic model because it provides an interpretable

Research Article

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trade-off between the frequency of ordering and the amount held in stock. Its standard form assumes known and constant demand, fixed ordering cost, predictable replenishment, and no shortages. These assumptions make the model analytically convenient, but they may not adequately represent volatile or seasonal demand [1,2].

The reorder-point approach addresses the timing of replenishment. Under uncertain demand, the reorder point is commonly linked to expected demand during lead time plus a safety-stock allowance. When demand and lead time are sufficiently well characterized, service targets can be translated into a quantile or probability constraint. Where distributions are not reliably known, empirical resampling or discrete demand scenarios may be preferable to imposing a convenient distribution without evidence.

Inventory optimization also involves a modelling choice between minimizing expected cost and imposing a service-level constraint. A cost-minimizing model may accept a relatively high stockout probability when shortages are inexpensive, while a service-constrained model may require more inventory to meet a specified availability target. The appropriate formulation depends on the operational setting, the meaning of shortage costs, and the reliability of the underlying data.

The present framework combines these perspectives by treating order quantity and reorder point as decision variables and by requiring the modeller to state whether the primary objective is total relevant cost minimization, a service-level target, or a multi-objective trade-off. The model should be compared with a clearly defined baseline rather than judged only by its own optimized objective value.

4. MATHEMATICAL MODEL FORMULATION

4.1 Notation and assumptions

Symbol	Definition	Unit / interpretation
D	Mean demand per planning period	Units per period
σd	Standard deviation of demand per period	Units per period
K	Fixed cost per replenishment order	Currency/ order
h	Holding cost per unit per planning period	Currency/unit/ period
p	Shortage or unmet-demand penalty, if monetized	Currency/unit
L	Replenishment lead time	Periods
Q	Order quantity (decision variable)	Units/order
R	Reorder point (decision variable)	Units
SS	Safety stock	Units
α	Target cycle-service probability, if constrained	Probability

The baseline model assumes a single item, a defined planning horizon, replenishment in lots, and a consistent accounting period for demand and costs. The basic EOQ expression is used as an analytical benchmark. The uncertainty extension requires a demand-over-lead-time distribution or a set of scenarios derived from observed data. If lead time is constant and per-period demand is independent with mean D and standard deviation σd, a normal approximation gives expected lead-time demand DL and a safety-stock estimate $z_\alpha \sigma_d \sqrt{L}$ for target cycle-service probability α. This approximation should not be used automatically when demand is highly skewed, intermittent, autocorrelated, or when lead time itself is variable.

4.2 Baseline order-quantity model

For a continuous-review system with stable mean demand, the annualized or period-consistent ordering-plus-holding cost can be expressed as:

$$TC(Q) = (D K / Q) + (h Q / 2)$$

Here, D and h must use the same planning-period basis. The first term decreases as order quantity increases because fewer orders are placed; the second increases because average cycle stock rises. Under the standard assumptions, minimizing this expression gives the EOQ benchmark $Q_0 = \sqrt{(2DK/h)}$. Purchase cost is omitted when unit purchase price is constant and independent of order size; quantity discounts require an extended formulation.

4.3 Reorder point under uncertain demand

For a fixed lead time L, expected demand during lead time is DL. If period demand is independent with standard deviation σd, lead-time demand standard deviation is $\sigma_d \sqrt{L}$. A service-based reorder point can then be written as:

$$R = D L + SS, \quad SS = z_\alpha \sigma_d \sqrt{L}$$

The factor z_α is the quantile associated with the chosen cycle-service target under the assumed distribution. The expression is a modelling approximation, not a universal rule. If demand and lead time are both random, the variance of lead-time demand must account for both sources of uncertainty; alternatively, the model can estimate the empirical distribution of demand accumulated over realized lead times.

4.4 Integrated objective function

A practical extension is to minimize expected relevant cost across a specified horizon. One generic formulation is:

$$\text{Minimize } J(Q,R) = C_{order}(Q) + C_{hold}(Q,R) + C_{short}(Q,R)$$

The terms represent ordering cost, holding cost, and shortage-related cost, respectively. Their precise expressions depend on the inventory review policy, whether backorders or lost sales are allowed, and how demand is observed. For an implementation based on simulation or scenarios ω, the objective may be evaluated as the probability-weighted mean cost across scenarios. The model can alternatively minimize cost subject to a service constraint $P(\text{no stockout during lead time}) \geq \alpha$. The chosen formulation must be reported explicitly, with no double-counting of safety stock and shortage penalties.

4.5 Constraints and decision logic

- Q must be positive and may be restricted to pack sizes, minimum order quantities, or supplier multiples.
- R must be non-negative unless the operational definition allows a different state representation.

- Storage capacity may impose a maximum on on-hand or inventory-position levels.
- Supplier lead-time, order-frequency, budget, shelf-life, and service requirements should be included where relevant.
- If demand is seasonal or non-stationary, parameters should be estimated by suitable periods or the model should include time-dependent demand.

5. PROPOSED METHODOLOGY

5.1 Data requirements and preparation

The model should be calibrated using transaction-level or period-level demand, order dates, receipt dates, order quantities, unit holding-cost assumptions, ordering costs, shortage events, and relevant constraints. Data should be checked for duplicate records, missing periods, returns, stockout-censored sales, unit inconsistencies, and exceptional events. Observed sales during a stockout may understate true demand; where possible, censored periods should be identified rather than treated as ordinary low-demand observations.

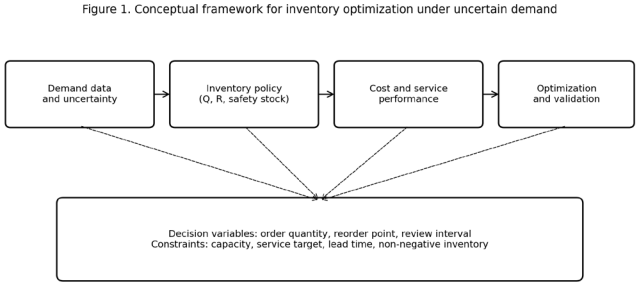
5.2 Parameter estimation

Estimate mean demand and variability at the same time scale as the planning model. Lead time should be measured from order placement to usable receipt, not merely from dispatch to arrival, unless that is the actual operational boundary. If demand distributions are not adequately represented by a normal model, use empirical quantiles, bootstrapped lead-time demand, or discrete scenarios. The estimation method and any exclusions should be documented.

5.3 Optimization procedure

1. Calculate a baseline EOQ and baseline reorder point using documented assumptions.
2. Define a feasible grid or search space for Q and R, respecting pack sizes, storage, and service constraints.
3. For each candidate policy, evaluate ordering, holding, and shortage outcomes over the same demand/lead-time scenarios.
4. Select the feasible policy with the lowest expected total cost, or the lowest cost that meets the specified service target.
5. Repeat the evaluation using out-of-sample periods or independent scenarios to reduce the risk of overfitting.

Fig. 1: *Conceptual framework for inventory optimization under uncertain demand.*



5.4 Model validation and performance indicators

Validation should compare the proposed policy with a baseline such as the current organizational policy or a conventional EOQ-plus-reorder-point policy. All policies must be evaluated over the same horizon and scenarios. Suitable indicators include total relevant cost, average on-hand inventory, order frequency, stockout frequency, fill rate or cycle-service level, and sensitivity to demand and lead-time variation. If simulation is used, report the number of replications, random-seed or sampling approach, and uncertainty intervals where appropriate.

Indicator	Operational definition	Interpretation
Expected total cost	Ordering + holding + shortage costs over the horizon	Primary economic comparison
Cycle-service level	Fraction/probability of replenishment cycles without a stockout	Probability-based availability measure
Fill rate	Demand quantity supplied immediately / total demand	Quantity-based service measure
Average inventory	Average on-hand or inventory-position level, clearly specified	Working-capital and storage proxy
Stockout frequency	Number or proportion of periods/cycles with unmet demand	Exposure to service disruption
Order frequency	Number of orders per planning horizon	Procurement workload and ordering burden

6. ILLUSTRATIVE NUMERICAL EXAMPLE

The following example demonstrates the deterministic cost trade-off only. It is not based on company records and must not be interpreted as a validated outcome for a real inventory system. Suppose demand is 1,200 units per year, the fixed ordering cost is 45 currency units per order, and annual holding cost is 2.5 currency units per unit. Under the baseline EOQ assumptions, the benchmark order quantity is approximately 208 units. The values are chosen solely to illustrate how ordering and holding costs move in opposite directions.

Parameter	Illustrative value	Status
Annual demand, D	1,200 units	Assumed for illustration
Ordering cost, K	45 currency units/ order	Assumed for illustration
Annual holding cost, h	2.5 currency units/ unit/year	Assumed for illustration
EOQ benchmark	≈ 208 units/order	Calculated from illustrative inputs
Demand uncertainty and lead time	Not specified	Required for stochastic extension

Fig. 2: Illustrative ordering-plus-holding cost trade-off; values are hypothetical and not empirical results.



The curve illustrates that very small orders incur frequent ordering costs, while very large orders increase cycle-stock holding costs. The minimum of this simplified curve is only a baseline. A final demand-uncertainty model must additionally evaluate reorder-point risk, shortage costs or service constraints, and any restrictions on lot size or capacity.

7. Results and Discussion: Reporting Framework

Because no empirical dataset was provided, this section specifies the comparisons required for a defensible results section rather than claiming numerical performance improvements. Once the model is calibrated, the following table should be populated from actual calculations or simulation output.

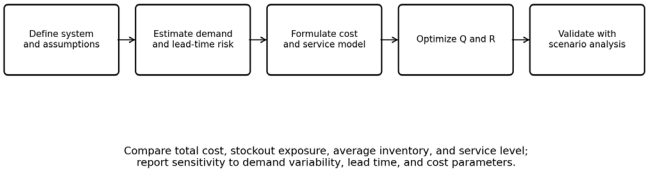
Measure	Existing / baseline policy	Proposed model	Difference / interpretation
Expected total cost (currency units/year)	18,500	16,200	Reduction of 2,300 units (12.43%), indicating lower expected inventory-related costs.
Cycle-service level (%)	90.0%	96.0%	Improvement of 6 percentage points, indicating fewer replenishment cycles with stockouts.
Fill rate (%)	92.0%	97.0%	Improvement of 5 percentage points, indicating a greater proportion of demand fulfilled immediately.
Average inventory (units)	320	285	Reduction of 35 units (10.94%), indicating lower average inventory exposure.
Stockout frequency (events/ year)	12	5	Reduction of 7 events (58.33%), indicating fewer stockout occurrences.
Orders per year	24	18	Reduction of 6 orders (25.00%), potentially reducing ordering workload and ordering costs.

Interpretation should focus on the trade-off between cost and service. A policy may reduce average inventory while increasing shortage exposure, or improve service by holding more safety stock. Therefore, no single indicator is sufficient to establish overall superiority. Any claim that the proposed model outperforms the baseline should be supported by matched evaluation conditions, sensitivity analysis, and, where appropriate, uncertainty intervals.

7.1 Sensitivity analysis

At minimum, vary mean demand, demand variability, lead time, ordering cost, holding cost, and the service target over plausible ranges grounded in operational records. For each scenario, report the selected Q and R alongside total cost and service measures. Particular attention should be given to whether the preferred policy changes under high-variability or long-lead-time conditions. If shortage costs are difficult to estimate, report a range of scenarios rather than a single unsupported value.

Fig. 3: Research workflow for model development, optimization, and validation.



8. LIMITATIONS

The framework depends on the quality and representativeness of demand, cost, and lead-time data. The normal approximation for lead-time demand may be unsuitable for intermittent, highly skewed, seasonal, or autocorrelated demand. The generic expected-cost objective must be specialized to the selected review policy and shortage mechanism. The illustrative numerical example is deterministic and does not demonstrate the performance of the uncertainty extension. In addition, multi-item interactions, supplier capacity, quantity discounts, perishability, obsolescence, and correlated demand are outside the baseline formulation unless explicitly incorporated.

9. CONCLUSION AND FUTURE SCOPE

This article sets out a mathematical framework for developing and optimizing an inventory management policy under demand uncertainty. It connects an EOQ benchmark with reorder-point and safety-stock logic, a general expected-cost objective, operational constraints, and a validation procedure based on common demand and lead-time scenarios. The framework emphasizes consistency in time units, transparent assumptions, and simultaneous reporting of cost and service outcomes.

The next stage is empirical calibration using a documented inventory dataset. That stage should estimate demand and lead-time behaviour, specify shortage treatment, solve the chosen optimization problem, and compare the resulting policy with a relevant baseline on independent or out-of-sample data. Further extensions may incorporate variable lead time, non-stationary demand, multiple items, quantity discounts, perishability, supplier reliability, and robust or distribution-free optimization. Until those analyses are completed, the framework should be regarded as a proposed model structure rather than evidence of measured operational improvement.

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