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EFFECT OF MECHANICAL PROPERTIES ON SPUR GEAR DYNAMICS

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ABSTRACT

The paper presents a dynamic tooth load analysis of spur gears with mesh stiffness. The analytical model is developed to simulate the load sharing characteristics through a mesh cycle. The model takes into account the main internal factors of dynamic load as time-varying mesh stiffness. The specific phenomenon of contact tooth pairs alternation during mesh cycle is integrated in this dynamic load modeling. A comparative study is included, which shows the effects of the mechanical properties on the dynamic behabiour of the system.

KEYWORDS: Spur Gear, Mesh Stiffness, Dynamic Loading, Contact Ratio.

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NOMENCLATURE

F- applied force,
φ-pressure angle,
G-shear modulus,
s-shear factor,
e_i and d_i -positions of the different discretized sections with sections area and inertia moments A_i and I_i;
E-Young modulus.
ν -Poisson ratio of the gear material.
b-face width, m-module,
Z-no. of teeth,
r_b-radius of base circle,
PCD-pitch circle diameter,
S_{ut} –ultimate tensile strength,
ρ -mass density.
m_e-equivalent mass of the gear pair,
c-damping co-efficient,
F_d-dynamic load,
ϵ-contact ratio,
F_n-Static load,
x_d-dynamic displacement

1. INTRODUCTION

Gear transmissions are widely used in several industrial applications. They are widely used in several sorts of machineries. Requirements for good running operations from these transmissions are low vibration and noise in addition to high efficiency. However, these requirements cannot be satisfied completely. Research on noise and vibration has revealed that the basic mechanism of noise generated from gearing is the gear box vibration excited by the dynamic load. The presence of defects [1] may also alter the normal operating conditions leading to higher vibration levels and a decrease in the efficiency of the transmission.

The dynamic analysis of the gears has become important part due to the pressing need of high speed and heavy load carrying machinery. In such applications, due to high precision, periodic variation of tooth stiffness is the only cause of noise and vibration. A high contact ratio spur gear pairs reduces the variation of tooth stiffness and thus to reduce vibration and noise. It also improves structural efficiency, reliability and power to weight ratio. High contact ratio gearing applies to gear meshes that have at least two pairs of teeth in contact at all times i.e. contact ratio of 2 or more. This helps in sharing of transmitted load. Unlike the low contact ratio spur gear drive, in high contact ratio spur gear drive the number of pairs of teeth in contact varies between two to three. In low contact ratio pair, the gear tooth is designed for critical loading condition, which corresponds to single pair of teeth contact and the load on gear tooth is maximum transmitted load. Since in a high contact ratio gear drives the minimum pairs in contact are more than one, the load sharing between the pairs to be calculated accurately for economical design of gear drive which will be helpful in design of high contact ratio spur gear drive.

Dynamic response prediction for gear teeth is a major consideration in gear design. In the present paper the dynamic behaviour of a gear pair will be analyzed in context of time varying load/speed conditions through a case study.

2. DYNAMIC MODEL

Assuming the gear system of the shafts and bearings are rigid, the sliding friction between the tooth surfaces of gear tooth is ignored. Fig.1

Research Paper

shows the dynamic model of gear pair. The teeth of meshing gears are considered as springs and the gear blanks as inertia masses. In developing this model, the dynamic process is studied in the rotating plane of the gears and the differential equations of motion are developed by using the theoretical line of action. In Figure 1, for a pair of contacting teeth, the time-varying mesh stiffness ke together with damping acts as parameter excitations.

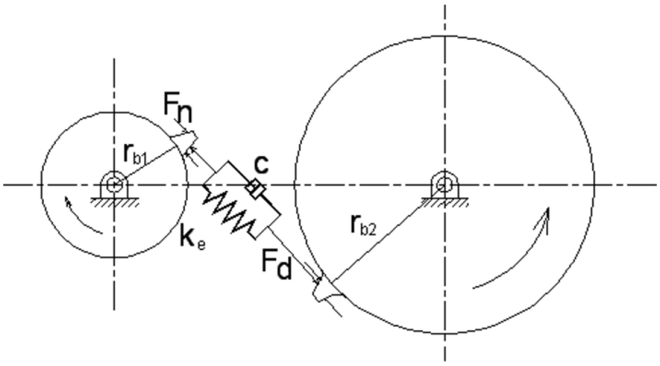


Figure 1: Dynamic model of meshing gears

If a tooth of a gear is loaded, the following types of deflection may occur:

(i) The first one is the bending deflection. The tooth is discretized into small section and the bending deflection in this case can be expressed as [1]:

$$\delta_b = F cos^2\varphi \sum_{i=1}^{n} e_i \left\{ \frac{(d_i - e_i d_i + \frac{1}{3} e_i^2)}{EI_i} + \frac{1}{sGA_i} + \frac{tan^2\varphi}{EA_i} \right\} \quad \ldots\ldots (1)$$

Notation:

F- applied force, φ-pressure angle, G-shear modulus, s-shear factor, e_i and d_i -positions of the different discretized sections with sections area and inertia moments A_i and I_i; E-Young modulus.

The corresponding bending stiffness of the tooth can be obtained by:

$$k_b = \frac{F}{\delta_b} \quad \ldots\ldots (2)$$

(ii) The second tooth deflection is that of the fillet- foundation of the tooth. It can be expressed by [2]:

$$\delta_f = \frac{F cos_\varphi^2}{bE} \left\{ L^* \left(\frac{u_f}{s_f}\right)^2 + M^* \left(\frac{u_f}{s_f}\right) + P^*(1 + Q^* tg^2\varphi) \right\} \quad \ldots\ldots (3)$$

Notation:

b-tooth width, the coefficients L*, M*, P*, Q* can be approached by polynomial functions depending on the geometrical characteristics of the gear.

The corresponding fillet-foundation stiffness can be obtained by:

$$k_f = \frac{F}{\delta_f} \quad \ldots\ldots (4)$$

(iii) Hertzian deflection is the third deflection occurring for a loaded tooth. It is generally a nonlinear function but can be linearized [3]. The corresponding Hertzian stiffness can be expressed by:

$$k_h = \left(\frac{\pi b E}{4(1-\nu^2)} \right) \quad \ldots\ldots (5)$$

where ν is the Poisson ratio of the gear material.

For a gear pair in contact the mesh stiffness (ke) can be obtained by putting in series the different stiffness of the gears:

$$k_e = 1 \Big/ \left(\frac{1}{k_{b1}} + \frac{1}{k_{b2}} + \frac{1}{k_{f1}} + \frac{1}{k_{f2}} + \frac{1}{k_h} \right) \quad \ldots\ldots (6)$$

where suffix 1 is for gear-1 and suffix 2 for gear-2.

During the engagement cycle, the contact load does not remain constant. This variation is mainly caused by the following factors:

(i) the alternating engagement of single and double pairs of teeth;

(ii) the variation of the mesh stiffness along the line of action;

(iii) the deviation of the tooth profile from the theoretical involute profile.

In order to build an accurate analytical model of the dynamic tooth load sharing, the parameters used in the model need to be estimated correctly.

The equation of motion of the gear pair in mesh is given by,

$$m_e \ddot{x}_d + c \dot{x}_d + \epsilon F_d = F_n \quad \ldots\ldots (7)$$

where $F_d = k_e x_d$

Notation:

me-equivalent mass of the gear pair, c-damping co-efficient, Fd-dynamic load involving elastic restoration only, ϵ-contact ratio, Fn-Static load, xd-dynamic displacement

The design parameters of the analyzed gear pair are chosen as (table-1 & 2):

Dimension	**Gear 1**	**Gear 2**
b	65 mm	65 mm
m	6.5 mm	6.5 mm
Z	25	98
rb	100 mm	200 mm
PCD	230 mm	430 mm
φ	20°	20°

Table 1: Dimensional parameters

Notation:

b-face width, m-module, Z-no. of teeth, rb-radius of base circle, PCD-pitch circle diameter, φ-pressure angle, Sut –ultimate tensile strength, E-elasticity modulus, G-rigidity modulus, υ -Poisson's ratio, ρ -mass density.

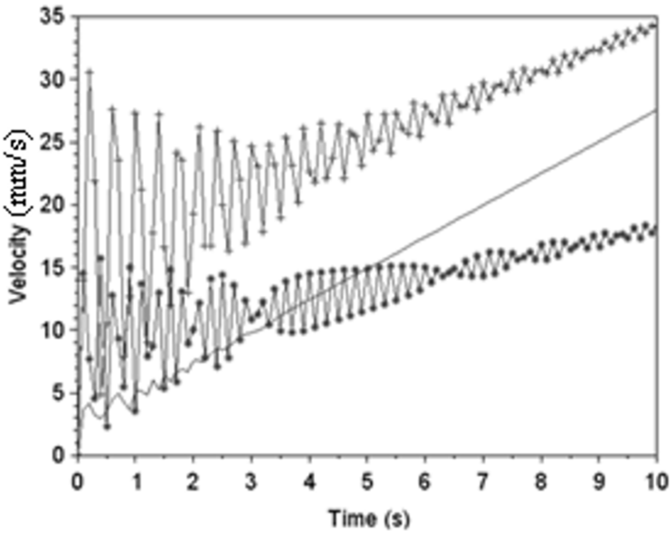
Property	Grey CI (FG 200)	Carbon Steel (40C8)	Stainless Steel (40Cr4Mo2)
S_{ut} (MPa)	200 MPa	600 MPa	800 MPa
E (GPa)	114 GPa	200 GPa	190 GPa
G (GPa)	46 GPa	80 GPa	77 GPa
υ	0.26	0.29	0.30
ρ (kg/m³)	7100	7850	7800

Table 2: Material properties

3. RESULTS AND DISCUSSIONS

Graphical solutions of the eqn.(7) are obtained in Scilab v.5.5.2. The material properties of three type of iron-based metals, viz. grey cast iron, carbon steel and stainless steel are considered for comparative study on dynamic behaviour. Amplitude of time-response is markedly decreased with the increase of tensile strength. Higher the strength lesser would be its vibration.

(a)



Legend: FG200: -----+
40C8: -----*
40Cr4Mo2:-------

(b)

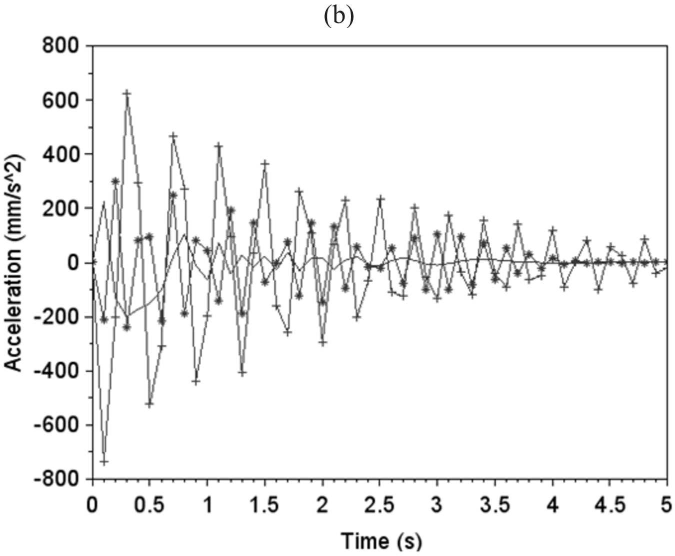


Figure 1: (a, b) Time responses of FG200, 40Cr8 & 40Cr4Mo2

4. CONCLUSIONS

A dynamic model of spur gears in mesh shows influence of mesh stiffness on their dynamic behavior. A comparative study is included, which shows the effects of the material properties of three type of iron-based metals, viz. grey cast iron, carbon steel and stainless steel on dynamic response. Dynamic response could be restrained within small value by proper selection of material. The study result offers a reference for gear system design. The solution to the problem also provides a reference for inherent characteristic and dynamic response for any gearbox transmission system.

5. REFERENCES

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6. APPENDIX-I

Equivalent mass of the gear pair:
$m_e = I_1 I_2/(I_1 r_{b2}^2 + I_2 r_{b1}^2)$,where I1, I2 are mass moment of inertia, rb1, rb2 are their base radii.
The damping coefficient can be calculated by
$c = 2\zeta\sqrt{m_e k_e}$
where ζ is the damping factor.
The meshing resonance frequency of the gear pair is determined as follows:
$\omega_n = \sqrt{\frac{k_e}{m_e}}$
Static load can be calculated from Lewis equation as follows,
$F_n = \pi y.b.m.\sigma_b$
Where b-face width, m-module, σ_b-bending stress,
y-Lewis form factor, Z-no. of teeth.
For 20° full depth, $y=0.154^{-\frac{0.912}{z}}$
Bending stress, $\sigma_b = S_{ut}/3$, where S_{ut} –ultimate tensile strength.