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ANALYTICAL STUDY OF RAMANUJAN'S INFINITE SERIES AND SPECIAL FUNCTIONS IN MODERN MATHEMATICS

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ABSTRACT

The mathematical discoveries of Srinivasa Ramanujan continue to occupy a central position in modern mathematical research due to their originality, depth, and far-reaching implications. Among his most remarkable contributions are his studies on infinite series, special functions, continued fractions, and asymptotic expansions. This paper presents an analytical study of Ramanujan's infinite series and special functions using contemporary mathematical approaches. The research focuses on selected identities from Ramanujan's notebooks and examines their mathematical structure, convergence properties, and theoretical significance.

The study reconstructs important results using methods from Complex Analysis, Real Analysis, and Number Theory. Special attention is given to Ramanujan's treatment of divergent series, gamma functions, zeta functions, and modular relations. The paper further explores how these results anticipated modern developments in asymptotic analysis and mathematical physics.

The findings reveal that Ramanujan's methods, though highly intuitive, align closely with advanced analytical frameworks developed later in the twentieth century. His infinite series transformations and special function identities continue to influence modern research in theoretical mathematics, quantum physics, and computational analysis. The study concludes that Ramanujan's work represents a unique synthesis of intuition and analytical depth, making his contributions enduringly relevant in contemporary mathematical sciences.

KEYWORDS: Ramanujan, Infinite Series, Special Functions, Gamma Function, Divergent Series, Asymptotic Analysis, Mathematical Identities

INTRODUCTION

The contributions of Srinivasa Ramanujan to mathematics remain among the most extraordinary achievements in the history of mathematical sciences. His notebooks contain an extensive collection of identities and formulas involving infinite series, continued fractions, modular equations, and special functions. Despite receiving limited formal mathematical training, Ramanujan developed highly sophisticated results that anticipated many concepts of modern mathematical analysis.

One of the central themes of Ramanujan's work is the manipulation and transformation of infinite series. He demonstrated exceptional ability in deriving rapidly convergent series, asymptotic expansions, and summation formulas. Many of these identities involve advanced mathematical structures such as the gamma function, theta functions, elliptic integrals, and the Riemann zeta function. His methods frequently relied on intuition and symbolic manipulation rather than rigorous proof, yet later mathematical investigations confirmed

the correctness of most of his discoveries.

Ramanujan also made pioneering contributions to special functions. His formulas involving the gamma function and modular equations provided important foundations for later developments in analytic number theory and complex analysis. In particular, his studies of divergent series and asymptotic approximations anticipated techniques that are now widely used in mathematical physics and quantum theory.

The notebooks of Ramanujan attracted the attention of distinguished mathematicians such as G. H. Hardy, who recognized the extraordinary originality of his work and collaborated in establishing rigorous proofs for many of his formulas. Today, Ramanujan's discoveries continue to inspire research in pure mathematics, cryptography, theoretical physics, and computational sciences.

This paper aims to analyze selected infinite series and special function identities from Ramanujan's work using modern mathematical frameworks and to evaluate their significance in contemporary mathematics.

OBJECTIVES OF THE STUDY

1. To analyze Ramanujan's contributions to infinite series and special functions
2. To examine selected mathematical identities using modern analytical methods
3. To reconstruct important formulas with formal mathematical proofs
4. To study the significance of divergent series and asymptotic expansions
5. To evaluate the influence of Ramanujan's work on modern mathematical research

MATHEMATICAL IDENTITIES AND PROOFS

1. Gamma Function Identity

Result

$$\Gamma(n+1) = n\Gamma(n)$$

Proof

Using the definition:

$$\Gamma(n) = \int_0^{\infty} x^{n-1} e^{-x} dx$$

Integrating by parts:

$$\Gamma(n+1) = \int_0^{\infty} x^n e^{-x} dx$$

Let:

$$u = x^n, \quad dv = e^{-x} dx$$

Then:

$$du = nx^{n-1} dx, \quad v = -e^{-x}$$

Therefore:

$$\Gamma(n+1) = n\Gamma(n)$$

2. Ramanujan's Infinite Series for π

Result

$$\frac{1}{\pi} = \frac{2\sqrt{2}}{9801} \sum_{k=0}^{\infty} \frac{(4k)!(1103+26390k)}{(k!)^4 396^{4k}}$$

Significance

This series converges extraordinarily fast and is widely used in high-precision computations of π

3. Zeta Function Identity

Result

$$\zeta(2) = \sum_{n=1}^{\infty} \frac{1}{n^2}$$

$$\zeta(2) = \frac{\pi^2}{6}$$

Proof (Outline)

Using Fourier series expansion of x^2 over $[-\pi, \pi]$, Euler established the result.

4. Divergent Series Interpretation

Result

$$1 + 2 + 3 + 4 + \dots = -\frac{1}{12}$$

Explanation

Using analytic continuation:

$$\zeta(-1) = -\frac{1}{12}$$

This interpretation appears in modern quantum field theory.

5. Asymptotic Partition Formula

Result

$$p(n) \sim \frac{1}{4n\sqrt{3}} e^{\pi\sqrt{\frac{2n}{3}}}$$

Significance

This formula, derived by Hardy and Ramanujan, revolutionized partition theory.

DISCUSSION

The mathematical identities examined in this study demonstrate the exceptional originality and analytical depth of Ramanujan's work. His manipulation of infinite series reveals an advanced understanding of convergence, transformation techniques, and symbolic relationships. Many of his results anticipated later developments in complex analysis and asymptotic theory.

Ramanujan's use of divergent series is particularly remarkable. Although such series were traditionally considered mathematically invalid, his intuitive interpretations closely resemble modern methods based on analytic continuation and zeta regularization. These techniques are now fundamental in several branches of theoretical physics.

The study also highlights Ramanujan's contributions to special functions, particularly the gamma and zeta functions. His formulas established important relationships between infinite series and transcendental functions, influencing later research in analytic number theory and modular forms.

Despite the absence of rigorous proofs in many cases, modern mathematical frameworks confirm the validity of a substantial portion of his results. This demonstrates the extraordinary power of Ramanujan's intuition and his ability to identify deep mathematical truths through unconventional reasoning.

FINDINGS

1. Ramanujan's infinite series identities are mathematically profound and highly original
2. His work on special functions significantly influenced modern analysis
3. Divergent series interpretations anticipated modern physical theories

4. Asymptotic expansions form a central feature of his mathematical methods
5. Modern analytical techniques validate many of his previously unproven results

CONCLUSION

The study confirms that Ramanujan's contributions to infinite series and special functions represent a landmark achievement in modern mathematics. His formulas, though often derived intuitively, possess remarkable analytical depth and continue to influence research across multiple disciplines.

Ramanujan's work serves as a bridge between classical analysis and contemporary mathematical theory. His discoveries involving gamma functions, zeta functions, asymptotic expansions, and divergent series remain foundational in modern mathematical research and theoretical physics.

The enduring relevance of Ramanujan's ideas demonstrates the timeless value of intuition in mathematical discovery. His notebooks continue to inspire mathematicians worldwide and remain an important source of innovation in modern analytical studies.

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