# PARTICLE SWARM OPTIMIZATION TO SOLVE CONTINUOUS TIME SUPPLY CHAIN PROBLEM

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**ABSTRACT**

The management of supply chain model with time varying depends on Particle Swarm Optimization (PSO) is presented. The discretization of state differential equations and control vectors is done by Runge Kutta method. The model is then converted to unconstrained optimization problem which is solved by PSO. The comparison between solution by PSO and optimal control theory is conducted.

**KEYWORDS:** Particle Swarm Optimization, Runge-Kutta Method, Optimal Control

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## 1. INTRODUCTION

Continuous time supply chain models involve the control of systems change with time. We published three researches focusing in supply chain models with continuous time and we solved them by optimal control systems [ 6] [7 ] [ 8] then we published another research paper solving it using genetic algorithm [13]. [4] Studied modern control theory concepts involving control policies. [5] The production Inventory system was studied. The lagrangian technique was used to solve the model. [9] The ideal production was identified to maximize the total profit associated with inventory and production levels. [14] The system of inventory control was considered and solved by optimal control. [3] New technique was introduced to solve online the continuous time linear quadratic regulator problem based on iterations. In our previous work we solved the SC models by control system approach but in this work we solve the model by PSO which considered as one of the artificial intelligence techniques. The comparisons between the solution of previous work (solution by optimal control approach) and the solution of current work (solution by PSO) is conducted.

Suppose the object functions in the form as:
Maximize or minimize $f(x)$
$$x = (x1, x2, x3, .......) \qquad (1)$$
Satyobroto Talukder (2011) introduced the following particle swarm algorithm.
1-Create population $n$ particles with stochastic positions $x_i^0$ and velocities $v_i^0$.
2-Put $t_{k+1} = t_K + 1$ For $k = 0,1,2,3....$
3-Calculate fitness $f(xi)$ for all particles
4- Set pi $= x_i$ if new position is better.
5-Identify global best position $G_{best}$ for each population.
6-Velocity and position is calculated by the following equation

$$v_i^{t+1} = v_i^t + c_1^t r_1^t [p_{best,i}^t - x_i^t] + c_2^t r_2^t [G_{best}^t - x_i^t] \qquad (2)$$

where
$v_i^t$ Particle velocity indexed $i$ at time $t$.

$x_i^t$ Particle velocity indexed $i$ at time $t$.

$p_{best\ i}^{\ t}$ Best position of particle indexed i existed from initialization through time t.

$G_{best}$ Global greatest $t$ position of any particle $i$ existed from initial case till time $t$.

c1 and c2 Constants used to level the contribution of the cognitive .

$r_1^t$ and $r_2^t$ Stochastic numbers flow uniform distribution U (0,1) at $t$.

7- When the positions for whole particles concur to approximately the same values, then the method has converged and the resultant value of $x_i^t$ the optimum solution else go to step 2.

## 2. NOTATIONS AND METHOD

We assume a company produces a particular product for the distributor, who sells them to the end consumer. Both company and distributor have on hand safety stock. Assuming the price is linear decreasing function with the sales. T : finishing time of plan; $u_{max}$: Maximum Vendor production. $h_1$:setup cost of seller company; $h_2$: Purchaser Setup cost *; c ;* fabrication cost *; a, b:* Coefficients used for the product in linear relationship between price and sales s(t) = a − b p(t)uˆ:Production goal level of the vendor ; x ; Inventory goal level of the vendor (safety stock) *; y ;* Inventory goal level of the buyer (safety stock) p[(t)] ;Price of one unit of the product at

time t (control variable) u(t) ; Production amount as function of time t. x ; Number of units in inventory of the buyer at time t. y : Number of units in inventory of the vendor at time t (state variable). We want the SC revenue to be maximized. The objective function of the model is as follow:

Max. J =
$$\int_0^T \left[ p(t)s(t) - \frac{1}{2} h_1 (x(t) - \hat{x})_1 - \frac{1}{2} h_2 (y(t) - \hat{y})_2 - \frac{c}{2} (u(t) - \hat{u})_1 \right] dt \quad (3)$$

**Subject to**

$$x = -s(t) \quad (4)$$

$$y = u_v(t) - s(t) \quad (5)$$

$$s(t) = a - bp(t) \qquad \forall\, t \in [0,T] \quad (6)$$

$$u(t) \le u_{\max} \qquad \forall\, t \in [0,T] \quad (7)$$

$$p(t) \le \frac{a(t)}{b(t)} \quad (8)$$

$$u(t),\ \hat{xt},\ y(t),\ s(t),\ \ge 0 \quad (9)$$

with initial conditions
x(0) = x , y(0) = y.

**2.1 Approximation by Runge-Kutta method**
To solve the above model described by equations (3),(4),(5),(6),(7),(8),(9), we must discretize the above derivative of variables x(t)and y(t ) by Runge method [1].
Substitute equation (6) into equation (4),(5) we have

$$x = -a + bp(t) \quad (10)$$

$$y = u(t) - a + bp(t) \quad (11)$$

Let h to be step size of the time and $t_i = t_{0+}ih$ then the flowing equations compute (10).

$$k_1 = hf(t_i, x_i) \quad (12)$$

$$k_2 = hf(t_i + \frac{h}{2}, x_i + \frac{k_1}{2}) \quad (13)$$

$$k_3 = hf(t_i + \frac{h}{2}, x_i + \frac{k_2}{2}) \quad (14)$$

$$k_4 = hf(t_i + h, x_i + k_3) \quad (15)$$

$$x_{i+1} = x_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) \quad (16)$$

Similarly we can do the same steps for $I_v$.

$$y_{i+1} = y_i + \frac{1}{6}(L_1 + 2L_2 + 2L_3 + L_4) \quad (17)$$

Also, replacing the integration symbol in continuous-time function (3) by the summation symbol, we get

$$\sum_{i=1}^{n} \left[ ap(t_i) - b(t_i)(p(t_i))_2 - \frac{1}{2} h_b \frac{(()-\hat{})_2}{I_b\ t_i\ I_b} - \frac{1}{2} h_v \frac{(()-\hat{})_2}{I_v\ t_i\ I_v} - \frac{c}{2} (u_v(t_i) - \hat{u}_v)_2 \right] h \quad (18)$$

**2.2 Implementation of Particle Swarm Optimization for solving Dynamic Supply Chain Model**
The following algorithm is explained the method of solution of SC problem.
1. Initialization; set initial time ( t0=0 ), set time step size h to reasonable value.

2.Set $t_{i+1} = t_i + h \ \forall\ i = 0,1,2,...n$
3.Approximate the state vectors derivative and control vectors derivative.

4.Remove the integration from objective function equation.

5.Solve the unconstrained optimization problem by PSO.

6.Stop if t = T (where T is end of time horizon) else go to step (2).
In PSO (using MAtLAB R2015b) there are many parameters that should be adjusted. The following parameters have to be selected, nvars; number of variables lb: lower bounds on search space, ub; upper bounds on search space, maxIter; number of generations.

**3. NUMERICAL EXAMPLE**
The solution method is illustrated by numerical example is .The model parameters are assumed to have the values as in table (1).

| Parameters | Value |
|---|---|
| c | 3$ |
| $h_1$ | 1$ |
| $h_2$ | 2$ |
| û | 15 |
| x̂ | 40 |
| ŷ | 25 |
| x(0) | 50 |
| y(0) | 15 |
| b | 0.5 |
| a | 30 |
| T | 5 |

**Table 1: Parameters of the model**

**3.1 Numerical Example Solution by Particle Swarm Optimization (PSO)**
In the current section, the proposed method is applied to solve dynamic supply chain problem by PSO. This can be achieved by the algorithm that is mentioned in section (2.2) using fourth order Runge-Kutta method. We set h = 0.5 from $t_i = 0$ to tn = 5 with step size h = 0.5 it takes 10 $t_0 = 0$, $t_1 = 0.5$, $t_2 = 1$, $t_3 = 1.5$, $t_4 = 2$, $t_5 = 2.5$, $t_6 = 3$, $t_7 = 3.5$, $t_8 = 4$, $t_9 = 4.5$, $t_{10} = 5$, i.e. i = 1, n =10. The problem is reduced to the following unconstrained problem which is solved by PSO. All computations are executed by particle swarm solver function existing in MATLAB R2015b. The results of objective function, inventory levels, price and production rate at end of planning horizon time i.e. ( T = 5) using both PSO and Optimal control Theory are showing in table (2).The solution of the example by optimal control approach was executed in paper research [ 6].

| The results | PSO | Optimal control |
|---|---|---|
| x5 | 7.80 | 7 |
| y5 | 31.82 | 30 |
| u (5) | 15.3 | 15 |
| p (5) | 30 | 30 |
| j | 1070 | 1064 |
| Execution time in seconds | 2.15 | 1.11 |

**Table 2: Summary of results t=T =5**

## 3.2 Comparison between Particle Swarm and Optimal Control

• The solution using optimal control theory is exact solution but the computations are hard. The solution using optimal control with help of MATLAB (m file) takes average time 1.11 second. The general solution of optimal control is hard

but it is suitable for continuous time system. The solution by optimal control theory does not have any approximation.

• The solution using PSO which is It is widely used to find the global optimum solution in a complex search space (Satyobroto Talukder 2011) is simple. The solution using PSO with help of MATLAB (particle swarm solver function) takes average time 2.15 second. The general solution of PSO is easy to implement but not suitable for continuous time system. The solution by PSO is an approximate solution resulting from conversion continuous time system to discrete time system.

## 4. Conclusion

This study has intended to apply PSO to find the solution for the problem of continuous time SC. The comparison between PSO solution and exact solution by optimal control has shown that the PSO solution is simpler than optimal control solution but it is not exact as optimal control. The model can be extended to embrace lot of buyers and lot of vendors and lot of products.

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