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TYPE II CHARGED GENERALISATION OF DURGAPAL'S SOLUTIONS FOR n ≥ 2

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ABSTRACT

We investigate the equilibrium configuration of a spherically symmetric charged fluid with a metric of the form $ds^2 = -e^{\lambda(r)}dr^2 - r^2(d\theta^2 + \sin^2\theta\, d\phi^2) + e^{\nu(r)}\,dt^2$. Using $e^\nu = A^2(1+x)^n$ where x= cr², the coupled Einstein-Maxwell equations are reformulated in terms of a first-order linear differential equation for $Z = e^{-\lambda}$. The general solution to this equation is derived, admitting integration constants and coefficients defined through recurrence relations. We explore a specific case n = 2 with and an electric filed $E^2 = \frac{\alpha^2 C_x}{2(1+x)}$ resulting in explicit expressions for the metric components, matter density ρ and pressure p.

INTRODUCTION

Regularity and physical plausibility criteria are discussed, ensuring ρ > 0, ρ > 0, and $\rho - 3p \geq 0$ throughout the fluid distribution. The solution transitions smoothly to the Reissner-Nordström metric at the boundary. Numerical estimates for mass, radius, and charge are provided, demonstrating compatibility with superdense stars. The model generalizes Durgapal's solutions for uncharged fluids and extends to higher n offering a framework for compact charged fluid distributions in equilibrium.

TYPE II CHARGED GENERALISATION OF DURGAPAL'S SOLUTIONS FOR N≥2

The space-time of spherical distribution of charged fluid in equilibrium is assumed to have the metric

$$ds^2 = -e^{\lambda(r)}dr^2 - r^2(d\theta^2 + \sin^2\theta\, d\phi^2) + e^{\nu(r)}dt^2 \quad (1)$$

We set

$$e^\nu = A^2(1+x)^n \quad (2)$$

where x = Cr² where C is a constant. The coupled Einstein-Maxwell equations on the background of this space-time are equivalent to,

$$8\pi\rho = \frac{e^{-\lambda}}{2}\left[\frac{\nu''}{2} + \frac{\nu'^2}{4} - \frac{\nu'\lambda'}{4} - \frac{\nu' + 5\lambda'}{2r}\right] \quad (3)$$

$$8\pi\rho = \frac{e^{-\lambda}}{2}\left[\frac{\nu''}{2} + \frac{\nu'^2}{4} - \frac{\nu'\lambda'}{4} - \frac{3\nu'\lambda'}{2r}\right] + \frac{e^{-\lambda} - 1}{r^2} \quad (4)$$

and

$$E^2 = \frac{e^{-\lambda}}{2}\left[\frac{\nu''}{2} + \frac{\nu'^2}{4} - \frac{\nu'\lambda'}{4} - \frac{\nu' + \lambda'}{2r}\right] + \frac{1 - e^{-\lambda}}{2r^2} \quad (5)$$

Choosing the new variable Z = e^{-λ} the equation (5) can be expressed as

$$\frac{dZ}{dx} + \frac{[(n^2 - 2n - 1)x^2 - 2x - 1]}{x(1+x)(1+(n+1)x)}Z = \frac{(1+x)\left[\frac{2E^2x}{C} - 1\right]}{x\,[1+(n+1)x]} \quad (6)$$

which is a first order ordinary linear differential equation. This differential equation can be solved formally by the usual method of solving a first order differential equation. The differential equation (6) admits the general solution,

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$$Ze^{\int p\,dx} = \int \frac{-(1+x)^{n-1}(1+(n+1)x)^{\frac{1-n}{1+n}}}{x^2}\,dx + \int \left[\frac{2E^2(1+x)^{n-1}(1+(n+1)x)^{\frac{1-n}{1+n}}}{Cx}\right] dx + k \quad (7)$$

where

$$e^{\int p\,dx} = \frac{(1+x)^{n-2}}{x}\,(1+(n+1)x)^{\frac{2}{n+1}} \quad (8)$$

and k is an arbitrary constant of integration. The part of the integral on the R.H.S of (2) not involving E^2 can be shown to have the form

$$\frac{(1+(n+1)x)^{\frac{2}{n+1}}\ f(x)}{x} \quad (9)$$

where

$$f(x) = a_0 + a_1 x + a_2\,x^2 + \cdots + a_{n-2}x^{n-2} \quad (10)$$

satisfies the differential equation

$$x(1+(n+1)x)\frac{df}{dx} - (1+(n-1)x)f(x) + (1+x)^{n-1} = 0 \quad (11)$$

When $n \geq 2$, the differential equation for f has the solution with

$$a_0 = 1 \quad (12)$$

$$a_{n-2} = \frac{-1}{n^2-2n-1} \quad (13)$$

and the coefficients with $0 < m - 1 < n - 2$ being determined by the recurrence relation

$$a_{m-1} = \frac{-1}{m(n+1)-2n}\left[(m-1)a_m + \frac{(n-1)(n-2)\ldots(n-m)}{m!}\right] \quad (14)$$

The integral involving E^2 on the R.H.S of (2) can only be evaluated when the form of the electric field intensity is specified. Durgapal's various solutions for spherical distribution of uncharged fluid at rest follow from the above solution on setting $E = 0$.

We discuss a specific model which follows from the above system by choosing $n = 2$ and $E^2 = \frac{\alpha^2 C_x}{2(1+x)}$ (15)
In this case the form of the solution (2) gives,

$$e^{-\lambda} = 1 + \frac{x}{(1+3x)^{\frac{2}{3}}}\int \alpha^2\,(1+3x)^{-1/3}\,dx + \frac{kx}{(1+3x)^{\frac{2}{3}}} \quad (16)$$

$$= 1 + \frac{kx}{(1+3x)^{\frac{2}{3}}} + \frac{\alpha^2 x}{2} \quad (17)$$

The matter density ρ and fluid pressure p of the distribution will have the explicit expressions

$$\frac{8\pi\rho}{C} = \frac{-\alpha^2}{2}\frac{(3+4x)}{(1+x)} - \frac{k(3+5x)}{(1+3x)^{\frac{5}{3}}} \quad (18)$$

$$\frac{8\pi\rho}{C} = \frac{4}{(1+x)} + \frac{k(1+5x)}{(1+3x)^{\frac{2}{3}}(1+x)} + \frac{\alpha^2}{2}\frac{(1+6x)}{(1+x)} \quad (19)$$

When $\alpha = 0$, the electric field vanishes, and the solution reduces to the one provided by Adler, which is a specific case of Durgapal's class of solutions with $n = 2$.

Physical Plausibility

For this solution to represent a feasible distribution of charged fluid in equilibrium, the expressions for E, ρ and p should be regular and further comply with the requirements $\rho > 0, p > 0 \; and \; \rho - 3p \geq 0$ throughout the region of validity. The expression (6) for E^2 indicates that E is well defined and regular at all $r, if \; C > 0$.

The matter density and fluid pressure assume the values

$$\frac{8\pi\rho(0)}{C} = \frac{-3\alpha^2}{2} - 3k \quad \text{and} \quad \frac{8\pi\rho(0)}{C} = 4 + k + \frac{\alpha^2}{2}$$

at the centre. The conditions $\rho > 0, p > 0 \; and \; \rho - 3p \geq 0$ will be fulfilled at the centre if and only if $-\left(4+\frac{\alpha^2}{2}\right) < k \leq -(2+\frac{\alpha^2}{2})$, provided C>0.

If the distribution extends up to a finite radius r = a, it is expected that the metric should continu- ously join with the exterior Reissner-Nordstorm metric,

$$ds^2 = -\left(1 - \frac{2m}{a} + \frac{q^2}{a^2}\right)^{-1} dr^2 - r^2\, d\theta^2 - r^2 \sin^2\theta\, d\phi^2 + \left(1 - \frac{2m}{a} + \frac{q^2}{a^2}\right) dt^2 \quad (20)$$

across the boundary $r = a$, where the fluid pressure vanishes. These conditions determine constants $k, A^2 \; and \; \frac{2m}{a}$ in terms of x_a and α^2 i.e,

$$k = \frac{-(1+3x_a)^{\frac{2}{3}}}{1+5x_a}\left[4 + \frac{\alpha^2}{2}(1+6x_a)\right] \quad (21)$$

$$A^2 = \frac{1}{(1+x_a)^2}\left[1 - \frac{x_a}{1+5x_a}\left(4+\frac{\alpha^2}{2}(1+7x_a)\right)\right. \quad (22)$$

and

$$\frac{2m}{a} = \frac{4x_a}{1+x_a} + \frac{\alpha^2 x_a^2(1+3x_a)}{(1+x_a)(1+5x_a)} \quad (23)$$

where

$$\frac{q^2}{a^2} = \frac{\alpha^2 x_a^2}{2(1+x_a)} \quad (24)$$

The estimates of the total mass m, the boundary radius a, the total charged q of a charged fluid sphere of this model together with the estimates of $A^2, C, \mu(\frac{\rho(a)}{\rho(0)})$ can be obtained in terms of the matter density ρ_a on the boundary and α^2 for various values of $x_a(= Ca^2)$.

These estimates for the charged fluid sphere with $\rho_a = 2 \times 10^{14} \; gm \; cm^{-3}$ and α^2=0.1 are reported in the table below.

x_a	A^2	C	μ	m	a	q
.05	.75	.0004	.85	1.05	11.09	.12
.15	.49	.0006	.66	4.06	15.54	.48
.25	.34	.0009	.55	6.88	17.13	.85
.35	.25	.0011	.47	9.34	17.89	1.2
.45	.19	.0013	.40	11.45	18.30	1.52
.55	.15	.0016	.36	13.28	18.53	1.83
.65	.12	.0019	.32	14.86	18.66	2.11
.75	.10	.0021	.29	16.26	18.73	2.37
.85	.08	.0024	.26	17.48	18.76	2.62

The variation of ρ,p and ρ – 3p against r is shown graphically in the following figure for the specific charged fluid sphere of this class with $x_a = 0.15$. The positivity of ρ,p and ρ- 3p throughout the fluid distribution is evident from this graph. The estimates for the mass and the size have the values clode to that of superdense stars in equilibrium. Similar studies for models with n = 3,4,5,... can be carried out. The Type II charged generalisations of Durgapal's solutions thus may be lead to physically viable models of compact charged fluids in equilibrium.

CONCLUDING REMARKS

I have presented a Type II charged generalization of Durgapal's solutions for spherically symmetric charged fluid distributions in equilibrium, extending the framework to higher values of n By refor- mulating the coupled Einstein-Maxwell equations, we derived explicit metric components, matter density, and pressure for a specific model with n = 2, ensuring regularity and physical plausibility throughout the fluid distribution. The solution smoothly transitions to the Reissner-Nordstr¨om metric at the boundary, satisfying the necessary junction conditions.

Numerical estimates for mass, radius, and charge demonstrate the compatibility of this model with the observed properties of superdense stars. The inclusion of an electric field introduces new degrees of freedom, offering insights into the structure and stability of compact charged fluid spheres. This framework generalizes earlier uncharged solutions and provides a basis for further exploration of charged models with higher n, which may yield additional physically viable configurations of compact stellar objects in equilibrium.

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